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WORMHOLES AND NEGATIVE ENERGY

J. J. Bevelacqua
Bevelacqua Resources
7531 Flint Crossing Circle SE.
Owens Cross Roads, AL 35763
bevelresou@aol.com

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Abstract

This paper investigates a possible wormhole and associated metrics, and the physical consequences of these geometries. Using the wormhole metric, the solution of Einstein's equation suggests that the energy density component of the energy-momentum tensor is negative. This result implies that a stable wormhole requires negative energy, and is consistent with Penrose's theory.

An evaluation of a representative set of spacetime geometries suggests that the occurrence of negative energy does not universally occur in every theoretical spacetime geometry. It does not occur in flat spacetime or in the Schwarzschild geometry. Negative energies theoretically occur under a limited set of conditions for a general static spherical geometry and the Friedmann-Robertson-Walker spacetime.

Keywords: General Relativity, Wormholes, Negative Energy, Exotic Astrophysical Structures

1.0 Introduction

Following Penrose, a stable wormhole requires the existence of negative energy [1]. It is important to emphasize the original concept of antiparticles and associated negative energies were proposed by Dirac [2]. The reader is also referred to a comprehensive treatment of the negative energy antiparticle concept by Santill [3].

The concept of negative energy has been associated with a number of physical effects. These include the Casimir effect [4], Lamb shift in hydrogen [(5) – (6)], anomalous magnetic moment of the electron [7], and Fulling-Davies-Unruh effect [(8) – (11)].

Associated with negative energy is the zero point energy concept. Related to these concepts are postulated physical effects including fluctuations that are typically quantified in terms of pair production and annihilation in a bubbling quantum foam, the zero-point energy of particle fields [(12)–(14)], virtual particle vacuum bubbles and loops [(15), (16)] , and interacting virtual particles [(17), (18)]. Although these theoretical arguments have been and continue to be made, there is no definitive experimental basis to select if any of these proposed mechanisms occur.

Other work emphasized the importance of negative energy in wormhole dynamics. Ida and Hayward [19] note that a traversable wormhole requires negative energy density, and argues how much negative energy is needed for wormholes. Their local analysis does not assume any symmetry, and allows dynamic (non-stationary) but non-degenerate wormholes. Davis [20] and Garattini [21] address traversable wormhole dynamics and the associated exotic energy requirements.

Of particular importance is the assertion of Penrose [1] that suggests the importance of negative energy in black hole formation, and the associated production of a stable wormhole. The negative energy requirement for wormhole stability [1] is the main objective of this paper. Morris and Thorne (MT) [22] provide additional commentary regarding wormhole stability.

This paper utilizes a set of spacetime geometries including the MT wormhole metric [22] to determine if negative energies occur. The associated Einstein tensor will be shown to exhibit a negative energy density for a subset of these metrics.

2.0 Wormhole Metric

Wormholes are theoretical constructs that have yet to be experimentally observed. The MT wormhole geometry can be represented by a spherically symmetric and time independent metric [(22), (23)]

$$ds^2 = -dt^2 + dr^2 + \left(b^2 + r^2\right)\left(d\theta^2 + \mathrm{Sin}^2\theta d\phi^2\right) \qquad (1)$$

where the constant b has the dimension of length.

The coordinates used to define the wormhole geometry are $\{t, r, \theta, \varphi\}$ where t is the time coordinate and (r, θ, φ) are spherical coordinates [23]. An examination of the wormhole geometry indicates that it reduces to flat spacetime in the limit $b \rightarrow 0$ [(22),(23)].

The MT wormhole geometry is a theoretical construct that is not based on experimental evidence. Except for the $b = 0$ metric, the geometry is not flat, but is curved. For $b \neq 0$, an embedding of the (r, φ) slice of the wormhole geometry produces a surface with two asymptotically flat regions connected by a region of minimum radius b [(22),(23)]. This region resembles a tunnel or wormhole connecting the two asymptotically flat regions [(24)- (26)].

The wormhole metric is time independent. It is also spherically symmetric because a surface of constant r and t has the geometry of a sphere. In addition, at very large r ($r \gg b$), the MT wormhole spacetime approaches flat spacetime [(22),(23)].

3.0 Einstein Equation

The Einstein equation can be expressed in the form

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad (2)$$

where $G_{\mu\nu}$ is the Einstein tensor that is a measure of the local spacetime curvature, and $T_{\mu\nu}$ is the energy-momentum tensor that characterizes the matter energy density. The T_{00} component is the energy density, T_{i0} and T_{0i} are the momentum density in the i direction, and T_{ij} are the i component of force per unit area exerted across a surface with a normal in direction j.

The Einstein equation will be used to demonstrate that negative energies result from the wormhole and other metrics. $G_{\mu\nu}$ is calculated and used to determine the coordinate basis components of $T_{\mu\nu}$.

4.0 Results and Discussion

Although the focus of this paper is the wormhole geometry, it is of interest to investigate other geometries to determine if the negative energy result is unique. These geometries include flat spacetime [(23) – (24)], the Schwarzschild geometry [(23), (24), (27), (28)], the Friedmann-Robertson-Walker (FRW) geometry [(23), (24), (28)], and a general static spherical geometry [(23), (24)].

4.1 Wormhole Geometry

The MT wormhole metric [22] was defined in (1). As noted in [23], the G_{00} component of the wormhole geometry is

$$G_{00} = -\frac{b^2}{\left(b^2 + r^2\right)^2} \tag{3}$$

(2) and (3) lead to an expression for the energy density

$$T_{00} = -\frac{1}{8\pi G}\frac{b^2}{\left(b^2 + r^2\right)^2} \tag{4}$$

T_{00} is the energy density that has a negative value for the MT wormhole geometry. Although the metric is a representation of the theoretical wormhole geometry, it provides an example that a static wormhole represented by (1) requires negative energy. This is consistent with Penrose's assertion [1], and these results can be interpreted to suggest that a wormhole requires negative energy to be stable.

It is of interest to investigate other geometries to determine if the negative energy result is unique. These geometries are addressed in subsequent discussion.

4.2 Flat Spacetime

For the flat spacetime geometry, the scalar curvature will be zero [23]. Zero curvature also suggests the associated tensors (e.g., the Ricci tensor and the Riemann curvature tensor) will also have few, if any, non-zero elements. In a similar fashion, the Einstein tensor in flat spacetime is expected to have few, if any, non-zero elements. This qualitative argument is supported by calculation of the elements of these tensors [23].

The flat spacetime metric has the form

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \operatorname{Sin}^2\theta \, d\phi^2 \tag{5}$$

As summarized in [23], the G_{00} component of the flat spacetime geometry is

$$G_{00} = 0 \tag{6}$$

(2) and (6) lead to an expression for the energy density that is zero. Accordingly, negative energies do not occur in the flat spacetime geometry.

4.3 Schwarzschild Geometry

The simplest curved spacetime geometries of general relativity are those that are the most symmetric. One of the most useful spacetime geometries is the Schwarzschild metric [(24), (27), (28)] that describes empty space outside a static, uncharged, spherically symmetric source of curvature (e.g., a spherical star of mass M that is uncharged and not rotating). In addition, the Schwarzschild geometry is a solution of the vacuum Einstein equation that describes spacetime devoid of matter.

The Schwarzschild metric has the form

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 + r^2 d\theta^2 + r^2 \operatorname{Sin}^2\theta \, d\phi^2 \tag{7}$$

As calculated in [23], the G_{00} component of the Schwarzschild geometry is

$$G_{00} = 0 \tag{8}$$

(2) and (8) lead to an expression for the energy density that is zero. Therefore, negative energies do not occur in the Schwarzschild geometry.

4.4 Friedmann-Robertson-Walker (FRW) Geometry

The FRW geometry [(24), (29)] describes the time evolution of a homogeneous, isotropic space that expands in time as the scale factor a(t) increases and contracts as a(t) decreases. The function a(t) contains information about the temporal evolution of the universe. In addition, a constant k is included in the FRW metric. The constant k determines the classification of the universe (i.e., k = +1 indicates a closed universe, k = 0 a flat universe, and k = −1 an open universe). Although the conventional terminology flat, closed, and open are used to distinguish the three possible homogeneous and isotropic geometries of spacetime, it is more physical to distinguish these features in terms of their spatial curvature [(23), (24)].

Homogeneity requires that the spatial curvature be the same at each point in the FRW geometries. The flat case has zero spatial curvature everywhere. The closed and open cases have constant positive and constant negative curvature, respectively.

The FRW metric has the form

$$ds^2 = -dt^2 + \frac{a^2(t)}{1-kr^2}dr^2 + r^2a^2(t)d\theta^2 + r^2a^2(t)\mathrm{Sin}^2\theta d\phi^2 \tag{9}$$

As summarized in [23], the G_{00} component of the FRW geometry is

$$G_{00} = \frac{3\left(k+\dot{a}^2(t)\right)}{a^2(t)} \tag{10}$$

(2) and (10) lead to an expression for the energy density

$$T_{00} = \frac{1}{8\pi G}\frac{3\left(k+\dot{a}(t)^2\right)}{a^2(t)} \tag{11}$$

Negative energy density occurs if $k+\dot{a}(t)^2<0$. For a flat universe with $k=0$ and $\dot{a}(t)^2\geq 0$, a negative energy density does not occur. Negative energy densities do not occur in a closed universe since $1+\dot{a}(t)^2\geq 0$.

In an open universe, negative energy densities can occur if $\dot{a}(t)^2<1$. Therefore, a negative energy density can only occur in an open universe under a limited set of conditions.

4.5 General Static Spherical Geometry

The rr and tt metric tensor elements of the general static spherical geometry are functions of r, namely exponential functions of $\lambda(r)$ and $\varphi(r)$. The corresponding metric tensor is given by

$$ds^2 = -e^{-2\phi(r)}dt^2+e^{-2\lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \mathrm{Sin}^2\theta d\phi^2 \quad (12)$$

In subsequent discussion, the derivative with respect to r is indicated by a prime. The general static spherical geometry reduces to the flat spacetime geometry in the limit of $\lambda(r)$ and $\varphi(r) \to 0$.

As summarized in [23], the G_{00} component of the general static spherical geometry is

$$G_{00} = \frac{1}{r^2}e^{-2\lambda(r)+2\phi(r)}\left(-1+e^{2\lambda(r)}+2r\lambda'(r)\right) \quad (13)$$

(2) and (13) lead to an expression for the energy density

$$T_{00} = \frac{1}{8\pi G}\left[\frac{1}{r^2}e^{-2\lambda(r)+2\phi(r)}\left(-1+e^{2\lambda(r)}+2r\lambda'(r)\right)\right] \quad (14)$$

In the general static spherical geometry, a negative energy density results if $-1+e^{2\lambda(r)}+2r\lambda'(r)<0$. Therefore, negative temperatures can occur under the special condition

$$\frac{d\lambda(r)}{dr} < \frac{1-e^{2\lambda(r)}}{2r} \quad (15)$$

Therefore, a negative energy density can occur in a general static spherical geometry under the selective condition noted in (15).

5.0 Wormhole Classifications

In subsequent discussion, we define four classes of theoretical wormholes. A conventional view of a wormhole in spacetime is a theoretical structure that connects two separate spacetime points A (t, r, θ, φ) and A' $(t', r', \theta', \varphi')$ [(1),(22),(23)]. These four classes can be illustrated in terms of a generalized line element

$$ds^2 = -d\tau^2 + dD^2 \tag{16}$$

where dD represents the spatial distance between A and A' for an arbitrary wormhole metric, and $d\tau$ is the corresponding temporal displacement. dD and $d\tau$ can be complicated functions of (t, r, θ, φ) to ensure stability of the wormhole. These functions would likely differ for each wormhole class and geometry. (16) is the basis for defining the four classes of wormholes.

In a Class I wormhole, there is a finite spatial distance between points A and A', and $d\tau$ is the travel time through the wormhole. Class I wormholes are structures that connect two spatial locations.

A second type of wormhole (Class II) connects points A and A' that are temporally separated. A Class II wormhole is characterized by no change in spatial location ($dD = 0$), but a difference in temporal coordinate ($d\tau \neq 0$). In this case, $d\tau$ is a change in the temporal coordinate that can be either positive or negative. A Class II wormhole connects the same spatial location to either the past or present relative to the entry point A into the wormhole. Theoretically, a Class II wormhole could permit a temporal displacement to the past or future.

Class III wormholes have $dD > 0$ and $d\tau \neq 0$. This classification permits the points A and A' to reside at different spacetime locations. A Class III wormhole allows a displacement to different spatial locations that could occur as either past ($d\tau < 0$) or future ($d\tau > 0$) events relative to the initial temporal coordinate.

A final classification (Class IV) is the null wormhole. Class IV wormholes have $dD = 0$ and $d\tau = 0$. In a Class IV event, no wormhole structure occurs and spacetime is not affected.

These four classes of wormholes outline areas for future research and the development of associated metrics. The validity of these theoretical classes and associated spacetime geometries must be confirmed by experimental observations.

6.0 Conclusions

A candidate wormhole metric is investigated, and its components are used to determine the energy density component of the energy momentum tensor (T_{00}). A negative T_{00} component of the energy momentum tensor suggests the existence of negative energy. Following Penrose, negative energy is required for the wormhole to be stable.

In general, negative energy does not universally occur. No negative energies are predicted for flat spacetime and the Schwarzschild geometry. In addition to the wormhole metric, negative energies are possible under limited conditions for the general static spherical and the closed Friedmann-Robertson-Walker geometries.

References

[1] R. Penrose, Gravitational Collapse and Space - Time Singularities, Phys. Rev. Lett. **14**, (1965) 57-59.

[2] P. A. M. Dirac, The Quantum Theory of the Electron, Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character **117**, (1928) 610-624.

[3] R. M. Santilli, Isodual Theory of Antimatter with Application to Antigravity, Grand Unification and Cosmology, Springer, A A Dordrecht, The Netherlands (2006).

[4] M. S. Morris, K. S. Thorne, and U. Yurtsever, Wormholes, Time Machines, and the Weak Energy Condition, Phys. Rev. Lett. **61**, (1988) 1446-1449.

[5] H. A. Bethe, The Electromagnetic Shift of Energy Levels, Phys. Rev. **72**, (1947) 339-341.

[6] S. L. Bu, Negative Energy: From Lamb Shift to Entanglement, arXiv:1605.08268v1 [physics.gen-ph] 18 May 2016 (2016).

[7] R. F. O'Connell, Effect of the Anomalous Magnetic Moment of the Electron on the Nonlinear Lagrangian of the Electromagnetic Field, Phys. Rev. **176**, (1968) 1433- 1437.
[8] S. A. Fulling, Nonuniqueness of Canonical Field Quantization in Riemannian Space-Time, Phys. Rev. **D7**, (1973) 2850-2862.
[9] W. G. Unruh, Notes on black-hole evaporation, Phys. Rev. **D14**, (1976) 870-892.
[10] P. C. W Davies, Scalar production in Schwarzschild and Rindler metrics, J. Phys. A: Math. Gen. **8**, (1975) 609-616.
[11] G. E. A. Matsas and D. A. T. Vanzella, The Fulling-Davies-Unruh Effect is Mandatory: The Proton's Testimony, arXiv:gr-qc/0205078v1 17 May 2002 (2002).
[12] J. J. Bevelacqua, Zero-Point Energy Conundrum, Hadronic Journal **46**, (2023) 305-313.
[13] J. J. Bevelacqua, Zero-Point Energy Conundrum for Spin ½ Particles, Qeios **5VZB0M**, (2023) 1-7. https://doi.org/10.32388/5VZB0M.
[14] J. J. Bevelacqua, Zero-Point Energy Conundrum for Spin 1 Particles, Qeios **8W0RRJ**, (2023) 1-7. https://doi.org/10.32388/8W0RRJ .
[15] J. J. Bevelacqua, Possible Planck-scale Physical Production Mechanisms and Consequences of Negative Energies – Initial Formulation, Physics Essays **34**(3), (2021) 342-351.
[16] J. J. Bevelacqua, Planck-Scale Quantum Field Theory –II-Evaluation of QED Analogue Amplitudes, Hadronic Journal **45**, (2022) 235-252.
[17] J. A. Wheeler, On the Nature of Quantum Geometrodynamics, Annal. Phys. **2**, (1967) 604-614.
[18] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, (1989) 1-23.
[19] D. Ida and S. A. Hayward, How much negative energy does a wormhole need?, Phys.Lett. **A260**, (1999) 175-181.
[20] E. W. Davis, Traversable Wormholes, Stargates, and Negative Energy, DIA-08-1004-004, Defense Intelligence Reference Document, Washington DC (2010).
[21] R. Garattini, Casimir wormholes, Eur. Phys. J. C (2019) **79:951**, 1-11.

[22] M. S. Morris and K. S. Thorne, Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity, Am. J. Phys. **56**, (1988) 395-412.
[23] J. J. Bevelacqua, Curvature Systematics in General Relativity, FIZIKA A **15**, (2006) 133–146.
[24] C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, W. H. Freeman and Company, San Francisco (1973).
[25] M. Visser, Lorentzian Wormholes: From Einstein to Hawking, Am. Inst. Phys., Woodbury, NY (1995).
[26] T. Müller, Visual Appearance of a Morris-Thorne-Wormhole, Am. J. Phys. **72**, (2004) 1045-1050.
[27] K. Schwarzschild, On the gravitational field of a mass point according to Einstein's theory I, Sitzber. Deut. Akad. Wiss. Berlin, Math.-Phys. Tech. **1916**, (1916) 189-196.
[28] K. Schwarzschild, On the gravitational field of a mass point according to Einstein's theory II, Sitzber. Deut. Akad. Wiss. Berlin, Math.-Phys. Tech. **1916**, (1916) 424-434.
[29] A. S. Friedmann, On the Curvature of space Z. Phys. **10**, (1922) 377-386.